Statistical data integration using multilevel models to predict employee compensation

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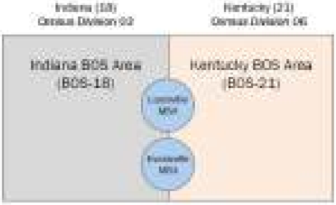
Contributions

Wage, benefits, and total employee compensation estimates

- Bureau of Labor Statistics (BLS)
- 242,686 domains defined as geography x occupation
  - metropolitan statistical areas (MSAs) and balance of state areas (BOSs); example:



  - 6-digit standard occupational classification codes (SOC6); example:
    - SOC2: 15-0000, Computer and Mathematical Occupations
    - SOC4: 15-2000, Mathematical Science Occupations
    - SOC6: 15-2041, Statisticians

Statistical data integration methodology

- Erciulescu A.L., Opsomer J.D., Schneider, B.J. (2021), "Statistical data integration using multilevel models." Under review.

Data

National Compensation Survey (NCS)

- wage and benefits survey estimates; in \$/hr
  - point estimates: $y_i^{NCS} = (y_{1,i}^{NCS}, y_{2,i}^{NCS})$
  - variance-covariance estimates, adjusted: Σ_i^{NCS}
  - levels: MSA/BOS/census division/nation x SOC6/SOC2/no SOC
  - variations: original scale, log scale, sum
- small sample

Occupational Employment Statistics (OES) program*

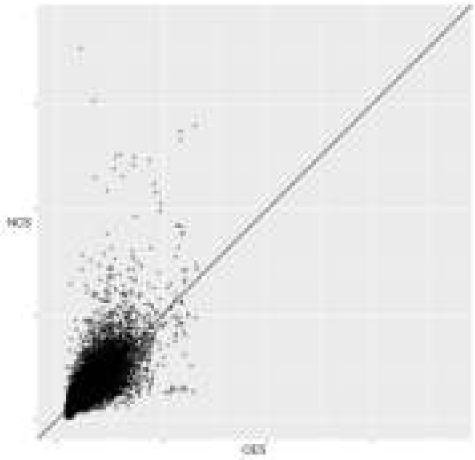
- wage survey estimates; in \$/hr
  - point estimates: $y_{1,i}^{OES}$
  - variance estimates, adjusted: $(\sigma_{1,i}^{OES})^2$
  - levels: MSA/BOS/census division/nation x SOC6/SOC2/no SOC
  - variations: original scale, log scale
- large sample

Prediction space: the set of domains for which there are sample data available in at least one of the two surveys; May 2019 as reference time

Occupational Employment and Wage Statistics Program - as of spring 2021

Need for data integration: distinct wage estimates

Domain-level wage survey estimates, MSA/BOS x SOC6



Two (large) domain-level NCS wage estimates were removed to improve visualization

Need for small area estimation: small sample data

Summary of sample sizes of domains in the prediction space, by level of aggregation; pseudo-effective sample sizes for NCS

Level	NCS			OES		
	Minimum	Median	Maximum	Minimum	Median	Maximum
MSA/BOS x SOC6	0	0	61	0	6	14,826
Census division x SOC6	0	1	191	1	236	68,810
Census division x SOC2	1	49	423	449	11,254	127,475
Nation x SOC6	0	8	796	21	2,272	366,362
Nation x SOC2	7	488	2,208	10,446	112,978	661,453

- median NCS sample size is 1 in NCS-only domains and 1 in all NCS domains
- median OES sample size is 5 in OES-only domains and 6 in all OES domains

Currently, BLS publishes employee compensation statistics at levels of aggregation defined using either geography or occupation (https://www.bls.gov/web/ecec/ececrse.htm).

Need for data integration and small area estimation: incomplete sample data

Number of domains in the prediction space, by level of aggregation

Level	Prediction Space Subset		
	NCS-only	NCS-and-OES	OES-only
MSA/BOS x SOC6	186	19,509	222,991
Census division x SOC6	0	4,358	2,565
Census division x SOC2	0	198	0
Nation x SOC6	0	721	50
Nation x SOC2	0	22	0

- small number of domains with benefits estimates
- large number of domains with two wage estimates
- very large number of domains with wage estimates from only one of the two sources

Hierarchical modeling estimation

Domain-level: MSA/BOS x SOC6-level survey estimates and associated variance estimates

- NCS-only domains (s_{NCS}), NCS-and-OES domains ($s_{NCS-OES}$), OES-only domains (s_{OES})

Bivariate: wage and benefits

- borrow strength from the strong relationship

Hierarchical Bayes: sampling levels, smoothing (latent) level, prior distributions

- borrow strength across surveys, across domains, and from covariates
  - covariates x_i defined in terms of area type (MSA or BOS), census division, and their two-way interactions
- link the NCS and OES wage estimates
- maintain the relationship between wage and benefits

Multi-fold: MSA/BOS x SOC6, SOC6

- borrow strength from the nested structure

Domain-level bivariate hierarchical Bayes multi-fold model

Sampling Level

$$
\begin{aligned}
y^{NCS}_{i,log}|(\theta_{i,log}, \Sigma^{NCS}_{i,log}) &\sim \mathrm{N}(\theta_{i,log}, \Sigma^{NCS}_{i,log}), i \in s_{NCS} \cup s_{NCS-OES} \\
y^{OES}_{1,i,log}|(\theta_{1,i,log}, \sigma^{OES}_{1,i,log}) &\sim \mathrm{N}\left(\theta_{1,i,log}, (\sigma^{OES}_{1,i,log})^2\right), i \in s_{OES} \cup s_{NCS-OES}
\end{aligned}
$$

Smoothing Level

$$
\begin{aligned}
\theta_{i,log}|(\beta, u_I, \Sigma_b) &\sim \mathrm{N}(x_i'\beta + u_I, \Sigma_b), i \in s_{NCS} \cup s_{NCS-OES} \cup s_{OES}, i \in I \\
u_I|\Sigma_u &\sim \mathrm{N}(0, \Sigma_u), i \in s_{NCS} \cup s_{NCS-OES} \cup s_{OES}, i \in I
\end{aligned}
$$

Prior Distributions

$$
\begin{aligned}
\beta &\sim \mathrm{N}(0, 10^4), \text{ component-wise} \\
(\Sigma_b, \Sigma_u) &\sim \text{inverse-Wishart}(I_2, 3), \text{ component-wise}
\end{aligned}
$$

- i indexes MSA/BOS x SOC6 domains
- I indexes SOC6 domains

Model fit, assumptions checks, prediction

Fit

- R JAGS
- Markov chain Monte Carlo (MCMC): 3 chains, 10,000 samples, 3,000 burn-in, thinning every 10th sample: 2,100 samples for inference
- SOC2-specific: 22 models

Assumptions checks

- MCMC diagnostics: $\hat{R}$, MC effective sample size, MC standard error, autocorrelation
- model specification: posterior predictive checks

Prediction

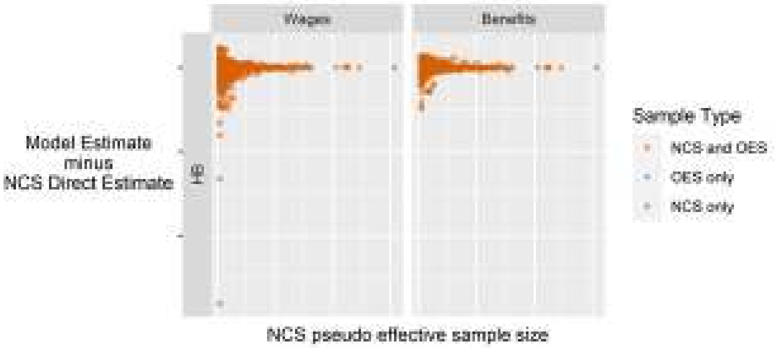
- posterior distribution

  $$[\theta_{i,log} | y^{NCS}_{log}, y^{OES}_{1,log}, \Sigma^{NCS}_{log}, \sigma^{OES}_{1,log}, x, \beta, \Sigma_b, \Sigma_u], i \in s_{NCS} \cup s_{NCS-OES} \cup s_{OES}$$

- transformations: exponential, sum

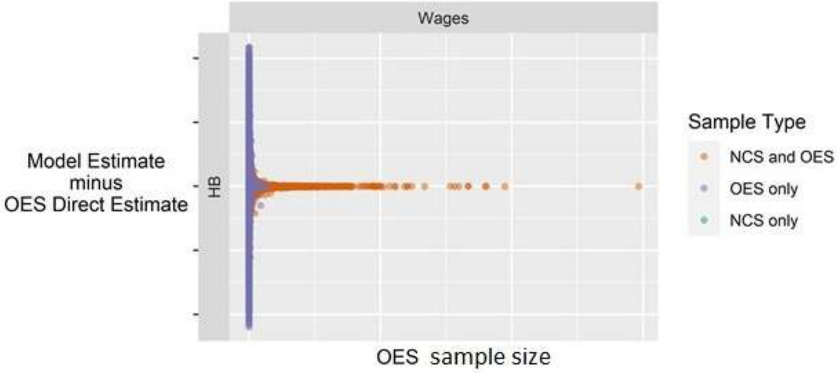
Comparison of NCS and model: point estimates

Domain-level wage and benefits estimates, MSA/BOS x SOC6



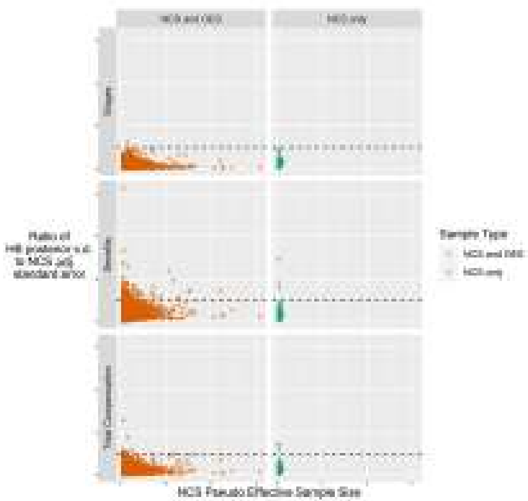
Comparison of OES and model: point estimates

Domain-level wage and benefits estimates, MSA/BOS x SOC6



Comparison of NCS and model: standard errors

Domain-level wage and benefits estimates, MSA/BOS x SOC6



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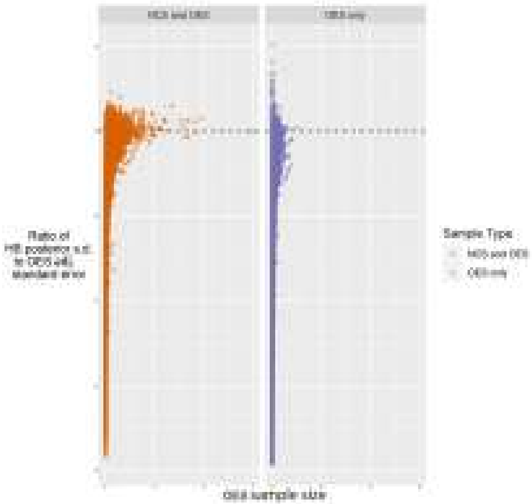

Comparison of NCS, OES, and model: coefficients of variation

Summary of coefficients of variation (%) of compensation estimates for the MSA/BOS x SOC6 domains in the prediction space

Estimation Approach	Wages		Benefits		Total Compensation	
	Median	$\% \geq 30$	Median	$\% \geq 30$	Median	$\% \geq 30$
Survey, NCS; adj. s.e.	49	77	90	92	58	83
Survey, OES; adj. s.e.	17	27	N/A	N/A	N/A	N/A
Model, HB	9	0	28	44	11	1

Recall there are 242,686 domains in the prediction space

Summary

- Methodological developments in statistical data integration, as extensions to small area estimation
- Incomplete survey data on two strongly-related variables
  - one variable collected on two surveys, the other collected only on the smaller survey
  - domains of interest represented by the union of the domains with sample data available for either variable and from either survey
- Complete set of wage, benefits, and total compensation estimates for all domains of interest, with associated uncertainty measures
  - granular levels lower than the levels at which current official statistics are available
- Hierarchical model estimates of improved precision, compared to the survey direct estimates

Selected references

Erciulescu, A.L., and J.D. Opsomer. 2019. "Task Order 5: Developing a Small Domain Estimation Methodology for the Office of Compensation and Working Conditions: Subtask 9: Final Report." Report Prepared for Bureau of Labor Statistics' Office of Compensation and Working Conditions.

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Myers, M., and D. A. Zamora. 2015. "Revisiting the Dilemma of Review for Modeled Wage Estimates by Job Characteristic." Monthly Labor Review, U.S. Bureau of Labor Statistics, September. https://doi.org/10.21916/mlr.2015.36.

Thank you!

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JAGS two-fold model specification

```
model{
  for(i in 1:mNCS){
        thetahatiNCS[i,1:C] ~ dmnorm(thetai12[i,1:C],  vhatdiriNCS.inv[i,1:C,1:C])
        vhatdiriNCS.inv[i,1:C,1:C] = inverse(vhatdiriNCS[i,1:C,1:C])
  }

  for(i in (mNCSO+1):m){
    thetahatiOES[i] ~ dnorm(thetai12[i,1],  vhatdiriOES.inv[i])
    vhatdiriOES.inv[i] = inverse(vhatdiriOES[i])
  }

  for(i in 1:m){
    thetai12[i,1] =  x1[i,1:P1]%*%beta1[1:P1] + v[i,1] + u[soc6s[i],1]
    thetai12[i,2] = x2[i,1:P2]%*%beta2[1:P2] + v[i,2] + u[soc6s[i],2]
    v[i,1:C] ~  dmnorm(muv[1:C], sigma2v.inv[1:C,1:C])
  }

  for (i in 1:msoc6s){
    u[i,1:C] ~  dmnorm(muu[1:C], sigma2u.inv[1:C,1:C])
  }

  ## priors:
  for (p in 1:P2){
      beta2[p] ~ dnorm(0, 1/100)
  }
  for (p in 1:P1){
    beta1[p] ~ dnorm(0, 1/100)
  }

  sigma2v.inv ~ dwish(Kv, 3)
  sigma2v = inverse(sigma2v.inv)

  sigma2u.inv ~ dwish(Ku, 3)
  sigma2u = inverse(sigma2u.inv)
}
```