Topics in Model-Assisted Point and Variance Estimation in Clustered Samples

By

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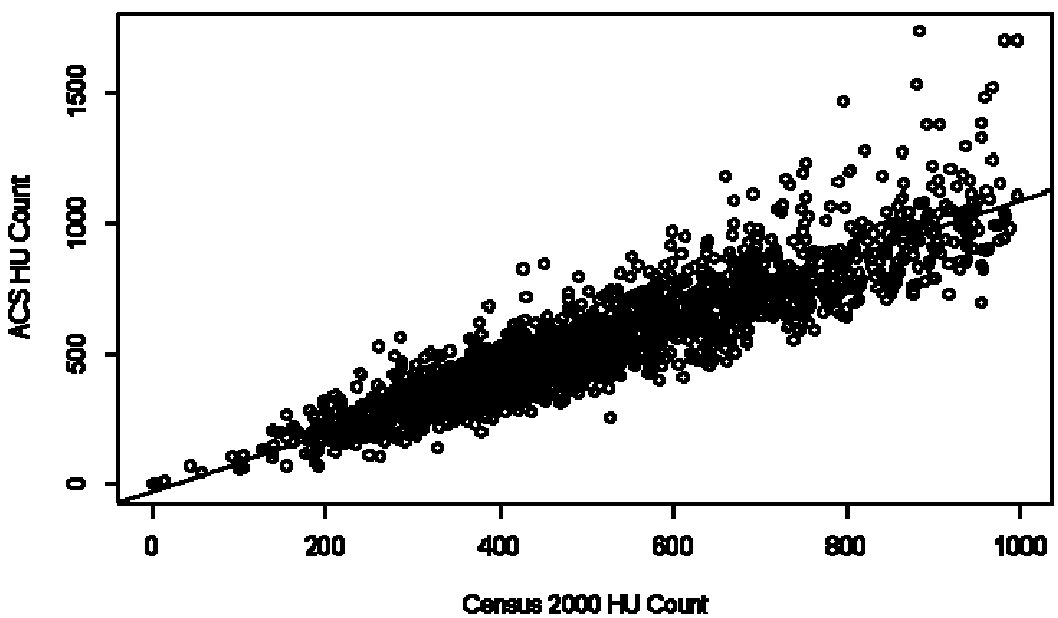
Outline

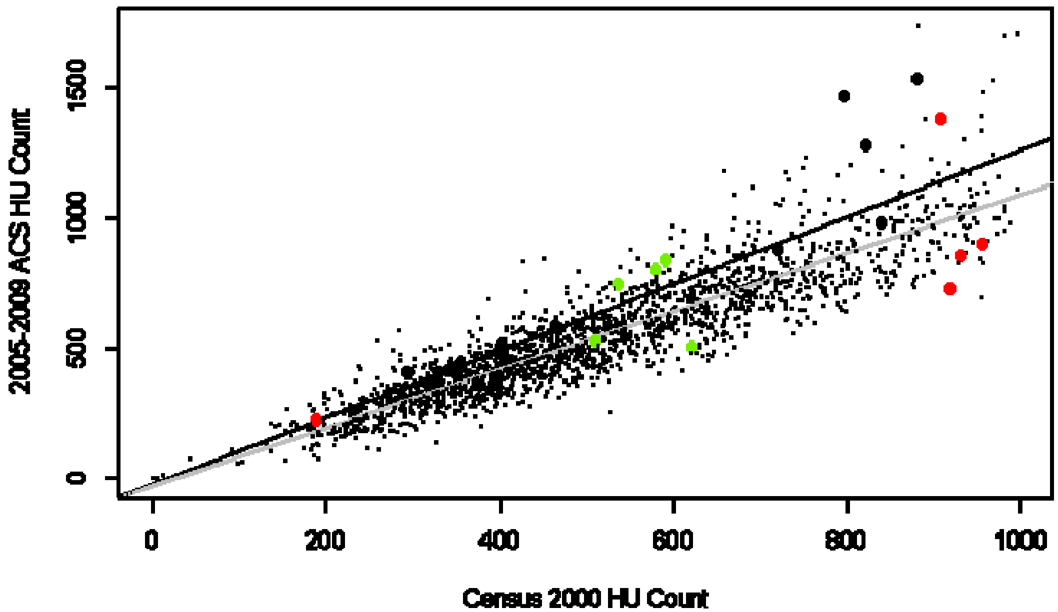
1. Improved Variance Estimators for Generalized Regression Estimators in Cluster Samples
2. Multivariate Logistic-Assisted Estimators of Totals from Clustered Survey Samples in the presence of Complete Auxiliary Information
3. Design-based Inference Assisted by Generalized Linear Models for Cluster Samples

Population



ACS HU Count
0
500
1000
1500
0
200
400
600
800
1000
Census 2000 HU Count

Sample Leverages



Estimator

- Generalized Regression Estimator (GREG)
  - $\hat{t}_y^{gr} = \sum_{\in \ U} \hat{y}_k + \sum_{\in \ s} d_k(y_k - \hat{y}_k)$
  - $var_M\left(\hat{t}_y^{gr}\right) = \sum_{\in \ s} \boldsymbol{g}_i^T \boldsymbol{\Pi}_i^{-1} \psi_i \boldsymbol{\Pi}_i^{-1} \boldsymbol{g}_i$

- Sandwich Variance Estimators
  - $v_R = \sum_{\in \ s} \boldsymbol{g}_i^T \boldsymbol{\Pi}_i^{-1} \boldsymbol{r}_i \boldsymbol{r}_i^T \boldsymbol{\Pi}_i^{-1} \boldsymbol{g}_i$
  - $v_D = \sum_{\in \ s} \boldsymbol{g}_i^T \boldsymbol{\Pi}_i^{-1} (\boldsymbol{I}_n - \boldsymbol{H}_{\boldsymbol{ii}})^{-1} \boldsymbol{r}_i \boldsymbol{r}_i^T \boldsymbol{\Pi}_i^{-1} \boldsymbol{g}_i$
  - $v_J = \sum_{\in \ s} \boldsymbol{g}_i^T \boldsymbol{\Pi}_i^{-1} (\boldsymbol{I}_n - \boldsymbol{H}_{\boldsymbol{ii}})^{-1} \boldsymbol{r}_i \boldsymbol{r}_i^T (\boldsymbol{I}_n - \boldsymbol{H}_{\boldsymbol{ii}})^{-1} \boldsymbol{\Pi}_i^{-1} \boldsymbol{g}_i$

Confidence Interval Coverage

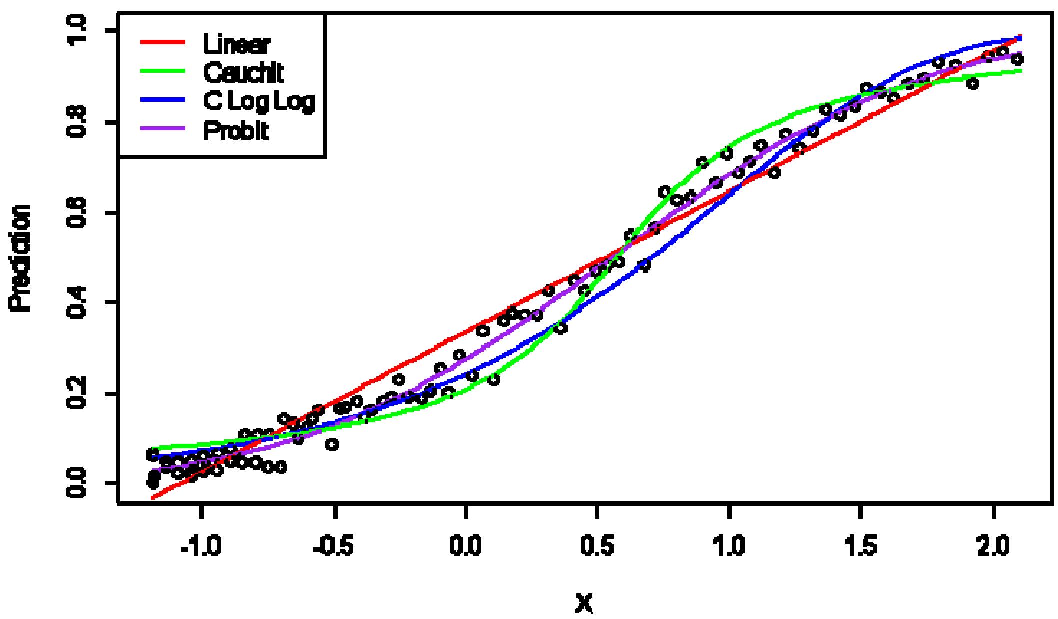
Estimator	**Lower**	**Middle**	**Upper**
Empirical	3.9	95.3	0.8
v_R	18.3	77.2	4.5
v_D	10.8	87.0	2.2
v_J	4.9	94.1	1.0

Conclusion of Leverage Adjusted Variance Estimators

- Small samples
  - Confidence interval coverage is closer to nominal value.
  - Central tendency (median) is closer to true value.
  - Extreme estimates are possible.
  - More variable.
- Large samples
  - Confidence interval coverage is closer to nominal value.
  - Conservative estimates.
  - Asymptotically unbiased.

Design-based Inference Assisted by Generalized Linear Models for Clustered Samples in the Presence of Complete Auxiliary Information

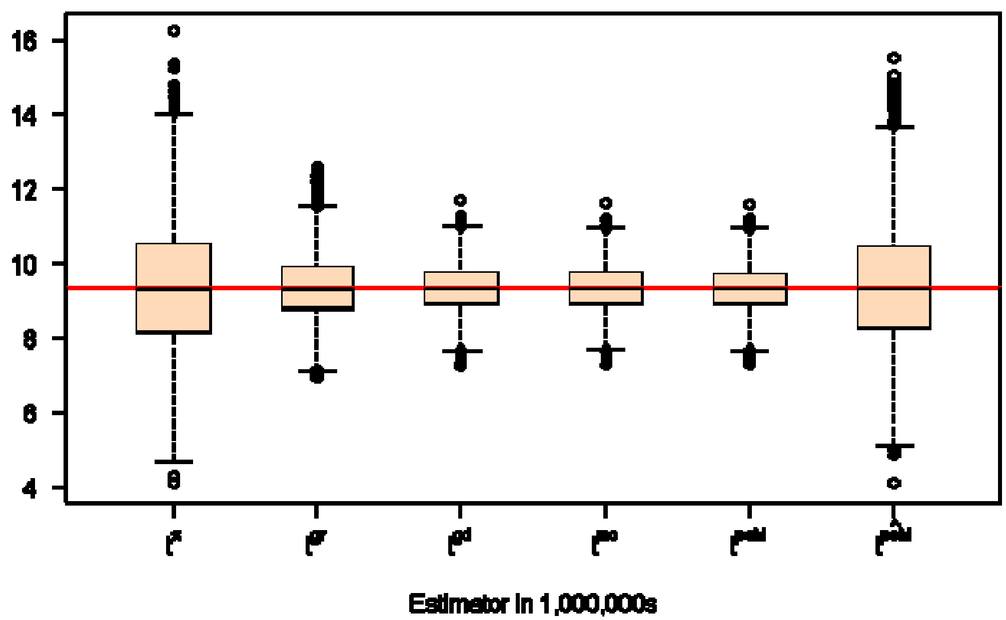
Example of a Binary Response from the 2000 Tract Level Planning Database



Estimators

- $\hat{t}_y^{\pi} = \sum_{\in\ s} d_k y_k$
- $\hat{t}_y^{pr} = \sum_{\in\ U} \hat{\mu}_k$
- $\hat{t}_y^{gr} = \sum_{\in\ U} \hat{y}_k + \sum_{\in\ s} d_k(y_k - \hat{y}_k)$
- $\hat{t}_y^{gd} = \sum_{\in\ U} \hat{\mu}_k + \sum_{\in\ s} d_k(y_k\ \ \mu\,\hat{}_k)$
- $\hat{t}_y^{mc} = \sum_{\in\ s} w_k^{mc} y_k$
- $\hat{t}_y^{peM} = M \sum_{\in\ s} p_k^{pe} y_k$
- $\hat{t}_y^{pe\hat{M}} = \hat{M} \sum_{\in\ s} p_k^{pe} y_k$

Box Plot of Logistic-Assisted Estimators of Renters in Large Samples



Results

- Calibrated estimators are asymptotically unbiased.
- Use canonical ink or calibrated estimators.
- Clear variance reductions of $\hat{t}_y^{gd}$, $\hat{t}_y^{mc}$, and $\hat{t}_y^{peM}$ over established estimators.
- GLM-assisted estimators require complete data.
- Estimators could be unstable in small samples.
- Performance of variance estimators depends on the sample design and sample size.

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